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A NEARFIELD MODEL OF THE PARAMETRIC RADIATOR.(U)

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Naval Underwater Systems Center
New London Laboratory
New London, Connecticut 06320

Technical Memorandum

A NEARFIELD MODEL OF THE PARAMETRIC RADIATOR

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LIST OF SYMBOLS

$S = \pi a^2$	=	Piston area
P_1, P_2	=	Primary pressures
p	=	Secondary (difference frequency) pressure
β	=	$1 + B/2A$ nonlinearity number ≈ 3.6 for water
ρ_0	=	Normal density
c_0	=	Normal sound speed
k_0	=	Secondary wavenumber
k_0	=	Geometric mean primary wavenumber
R_0	=	Geometric mean primary Rayleigh distance
\vec{r}_0	=	Range (vector)
L	=	Range (scalar)
θ	=	Angle to field point
2α	=	$\alpha_1 + \alpha_2$ = Absorption (nepers/m)
x, t	=	Integration range variables
$2\alpha L$	=	Scaled range
$2\alpha R_0$	=	Absorption number
$2\alpha R_0 k/k_0$	=	Nearfield absorption number
$2\alpha R_0 k_0/k$	=	Farfield absorption number
k_0/k	=	Downshift ratio
W_0	=	Westervelt source level
Y	=	PL/W where PL is the apparent source level at L
z	=	$u + iv$ complex variable
θ_w	=	$\sqrt{4\alpha/k}$ = Westervelt angle
θ/θ_w	=	

ABSTRACT

It is often necessary to measure and operate parametric radiators under conditions where finite range effects are important. A nearfield model is presented in which the source level is obtained by assuming the primary waves to be plane-cylindrical or diverging-conical depending upon whether the observation range is less or greater than the Rayleigh distance. Asymptotic beam patterns are obtained from the solution of an exponentially tapered line array. Nonlinear attenuation is not considered.

ADMINISTRATIVE INFORMATION

This memorandum was prepared under Project A-614-15, Subproject 11-121-706 Task 17442 "Parametric Echo Ranging Systems", Principal Investigator W. L. Konrad, Sponsoring activity NSSC SEA 06H1 J. Neely.

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INTRODUCTION

In the Westervelt¹ model of the parametric radiator, an asymptotic approximation was made to obtain the source level and beam pattern at very long ranges. In many practical cases the conditions for this model are not met, therefore a finite-range model is essential.

Several methods²⁻⁷ have been employed to obtain the field at finite range using various simplifying assumptions. In the present method the asymptotic beam pattern for points sufficiently far outside the primary beam is obtained from the solution for an infinite exponentially-tapered line array. The source levels are obtained by assuming the primary waves to be plane-cylindrical or diverging-conical depending on whether the observation point is less or greater than the Rayleigh distance. The other assumptions are that the absorption is linear and small compared to the secondary wave number.

Beam Patterns of a Line Array

1 For a parametric array the source strength per unit length is given by

$$q(x,t) = \beta S \partial/\partial_t (P_1 P_2) / \rho_0^2 c_0^4 \quad (1)$$

where $P_n = P_{on} \exp [-x(\alpha_n + ik_n) + i\omega_n t]$, $n = 1, 2$

The difference frequency pressure for small S becomes

$$P(\vec{r}, t) = -\rho_0 \int_0^\infty dx q'(x, t - |\vec{r} - x|/c_0) / 4\pi |\vec{r} - x| \quad (2)$$

where q' indicates the time derivative

For $\vec{r} \rightarrow \infty$ Eq (2) gives the Westervelt result

$$rP(\theta) = \beta P_1 P_2 k^2 S / 8\pi \rho_0 c_0^2 \left[\alpha + ik \sin^2(\theta/2) \right]^2 \quad (3)$$

and $W = rP(0)$ is the Westervelt source level.

(The term $\exp(i\omega t - ikL)$ is omitted throughout.)

For finite range Eq. (2) can be written in terms of the dimensionless variable $Y = PL/W$ according to the geometry of Fig. 1 as

$$Y = 2\alpha L \exp(2\alpha L) \int_{-L}^{\infty} dx \frac{\exp[-2\alpha x - ik(\sqrt{x^2 + a^2} + x)]}{\sqrt{x^2 + a^2}} \quad (4)$$

Eq. (4) can be rewritten as a contour integral

$$Y = 2\alpha L \exp(2\alpha L) \int_{z_0}^{\infty} \frac{dz \exp(-z)}{\sqrt{z^2 + B^2}}$$

where

$$z = 2\alpha x + ik(\sqrt{x^2 + a^2} + x) \quad (5)$$

$$z_0 \approx -2\alpha L + ika^2/2L$$

$$B = 2a (\alpha^2 + ik\alpha)^{1/2}$$

and a typical contour is shown as the upper curve of figure 2.

Since $k/\alpha \gg 1$ is the domain of interest the a^2 term in B will be neglected. Further, for small angles it is convenient to normalize $\theta \approx a/L$ to the Westervelt angle θ_w , hence,

$$z_0 = -u_0 + iv_0 \approx 2\alpha L(-1 + i\theta^2)$$

$$B^2 \approx 4i(2\alpha L\theta)^2 = 4iu_0v_0$$

where $\theta = \theta/\theta_w$.

Since the path of integration becomes rapidly oscillating for large z , the lower path of Fig. 2 where $v = v_0$ may be substituted to give the same result.

Changing variables to $z = t - u_0 + v_0$. Eq. (5) becomes

$$Y = 2\alpha L \exp(-iv_0) \int_0^{\infty} \frac{dt \exp(-t)}{\sqrt{(t - u_0)^2 - v_0^2 + 2iv_0(t + u_0)}} \quad (6)$$

Eq. (6) was evaluated numerically and the results are shown in Figs. 3 and 4. Note that for the line source ($S \rightarrow 0$) there is a logarithmic singularity of strength $\exp(-2\alpha L)$ as $\theta \rightarrow 0$. Only for values of $2\alpha L > 10$ do patterns fall within 1 dB of the Westervelt asymptote. For smaller values of $2\alpha L$ the patterns tend to become narrower, approaching a minimum at $\theta = 1$ and increasing thereafter.

The line array patterns are asymptotic patterns of a parametric radiator at points sufficiently far outside the primary beams. For experimental comparison the axial level must be known. In order to determine the axial level it is necessary to account for the finite cross-section specifically.

CYLINDRICAL SOURCE

For a plane wave cylindrical source of radius a and the field point on the axis we have

$$P = \frac{\beta P_1 P_2 k^2 \exp(2\alpha L)}{4 \pi \rho_0 c_0^2} \int_{-L}^{\infty} dx \int_0^a da \frac{2 \pi a \exp[-2\alpha x - ik(\sqrt{x^2 + a^2} + x)]}{\sqrt{x^2 + a^2}} \quad (7)$$

Integrating over a we have

$$P = \frac{\beta P_1 P_2 k^2}{4 \pi \rho_0 c_0^2} \exp(2\alpha L) \left[\int_{-L}^0 dx \exp(-2\alpha x) + \int_0^{\infty} dx \exp(-2\alpha x - 2ikx) + \int_{-L}^{\infty} dx \exp(-2\alpha x + ik(\sqrt{x^2 + a^2} + x)) \right] \quad (8)$$

The second integral is omitted since $k \gg \alpha$ and its contribution negligible. Taking $z = 2\alpha x + ik(\sqrt{x^2 + a^2} + x)$ in the third integral and neglecting terms in a^2 we obtain

$$4\alpha x = z - \sqrt{z^2 + B^2}$$

from which

$$4\alpha dx = dz \left[1 - \frac{z}{\sqrt{z^2 + B^2}} \right]$$

Substitution of the modified contour $z = t - u_0 + iv_0$ in the third integral gives finally

$$y_0 = \frac{1}{iv_0} \left[\int_0^{u_0} dt \exp(-t) - \frac{1}{2} \exp(-iv_0) \int_0^{\infty} dt \exp(-t) \left(1 - \frac{t - u_0 + iv_0}{\sqrt{(t - u_0)^2 - v_0^2 + 2iv_0(t + u_0)}} \right) \right] \quad (9)$$

where $v_0 = ka^2/2L \approx 2\alpha R_0 k/k_0 u_0$.

Eq. (9) was evaluated as a discontinuous function at $t = u_0$, since the integral converges more rapidly than by integrating the terms separately.

The results shown in Fig. 5 give the apparent source level as a function of scaled distance referred to the Westervelt source level at infinity. The parameter is the nearfield absorption number. Oscillations occur as the source level vs range changes from square-law to a slower rate. For all values of the parameter the asymptotic approach for the Westervelt limit is from above. This effect has a simple geometric explanation, i.e., the distance is measured from the origin rather than the "center of gravity" of the array, hence apparent source levels tend to be higher.

Note that the effects of absorption are negligible for $2\alpha L < 0.1$. From $2\alpha L = 0.1$ to 1 absorption begins to have an effect and for $2\alpha L > 1$ The curves approach the Westervelt limit. The resulting curves are asymptotic for $L \ll R_0$.

CONICAL SOURCE

For $L \gg R_0$ consider a conical source of spherically diverging primary waves and field point on the axis. Eq. (7) becomes

$$P = \frac{P_1 P_2 k^2 R_0^2}{2\rho_0 c_0^2} \int_0^{\theta_0} d\theta \sin\theta \int_0^{\infty} dr \frac{e^{-[2\alpha r + ik(\sqrt{r^2 + L^2} - 2Lr\cos\theta + r - L)]}}{\sqrt{r^2 + L^2} - 2Lr\cos\theta} \quad (10)$$

Integrating over θ we obtain

$$p = \frac{P_1 P_2 k R_0^2 \theta_0^2}{12 \rho_0 c_0^2 L} \left[\int_0^L dr \frac{\exp(-2ar)}{r} + \int_L^\infty \frac{dr \exp[-2ar - 2ik(r-L)]}{r} - \int_0^\infty dr \frac{\exp[-2ar - ik(\sqrt{r^2 + L^2 - 2Lr\cos\theta_0} + r - L)]}{r} \right] \quad (11)$$

The second integral is again omitted. Substituting z for x in the third integral as in the cylindrical case and taking the new contour we have after some manipulation

$$Y = \frac{u_0}{iv_0} \left[\text{Res.} + \int_0^{u_0} dt \frac{\exp(-t)}{t} - \int_0^\infty dt \frac{\exp(-t)}{t + u_0 + iv_0} \frac{1 - \frac{t - u_0 + iv_0}{\sqrt{(t - u_0)^2 - v_0^2 + 2iv_0(t + u_0)}}}{\sqrt{(t - u_0)^2 - v_0^2 + 2iv_0(t + u_0)}} \right] \quad (12)$$

where $v_0 = k L \theta_0^2/2 \approx u_0/(2 \ll R_0 k_0/k)$ and Res. is the residue at z_0 .

Although each integral has a pole at $t = 0$, the sum is well-behaved. Near zero the last integrand of Eq. (11) behaves as $1/z$, where $z' = (1 + iv_0/u_0)t$ and it is necessary to evaluate the residue between the two paths. Take the contour from $z' = (1 + iv_0/u_0)\epsilon$ to $z' = (1 + i0)\epsilon$ and let $\epsilon \rightarrow 0$, which gives

$$\int_C \frac{dz'}{z'} = -\frac{1}{2} \ln [1 + (v_0/u_0)^2] - i \tan^{-1}(v_0/u_0)$$

Using the residue as the initial value, Eq. (12) was integrated as before and the results are shown in Fig. 6. The parameter in this case is the farfield absorption number. For values greater than 1, the approach to the asymptotic is again from above. For values less than 1, the asymptotic source levels fall below the Westervelt asymptotic and approach the asymptotic from below. This is the farfield diffraction effect where the generation becomes spherical and the source level approaches the exponential integral rather than the Westervelt asymptote.

COMPARISON WITH EXPERIMENT

Figure 7 shows the predicted source levels vs range compared to experimental results⁵. The downshift ratio was 7 and the value of $2\alpha R$ was 0.0175. The cylindrical and conical curves intersect near the Rayleigh distance. Above this point agreement with the conical curve is excellent while below the deviations are significant.

Figure 8 shows a comparison of experimental and predicted source levels vs range for a downshift of 50 with $2\alpha R = 0.17$. Experimental values are approximately 3 dB low near the Rayleigh distance.

Figure 9 presents beam patterns of the second example made at two different distances: $2\alpha L = 0.25$ and 1. In the first case the experimental, rather than predicted axial value was used.

CONCLUSIONS

Predictions based on the nearfield model appear to be in fair agreement with experiment, however, further comparisons must be made to determine all the conditions for validity since errors can be large near the Rayleigh distance. The particular problem of nonlinear attenuation must also be included if the model is useful in a practical sense.⁸ In its present form, however, it serves to illustrate the various finite range phenomenon of an absorption limited parametric radiator.

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FIGURE CAPTIONS

- Figure 1 Geometry of Line Source
- Figure 2 Contour Integration Paths. Branch points lie outside both paths and are not involved.
- Figure 3 Beam Patterns. θ is the angle referred to the Westervelt angle and the parameter is the scaled distance $2\alpha L = 10$ to 1
- Figure 4 Beam patterns $2\alpha L = 1$ to .01
- Figure 5 Source level relative to Westervelt vs scaled distance. The parameter is the nearfield absorption number (plane wave model).
- Figure 6 Source level relative to Westervelt vs scaled distance. The parameter is the farfield absorption number (spherical wave model).
- Figure 7 Comparison of predicted and experimental values of apparent source level vs range $2\alpha R_0 = 0.0175$, $k_0/k = 7$
- Figure 8 Comparison of predicted and experimental values of apparent source level vs range $2\alpha R_0 = 0.17$, $k_0/k = 50$.
- Figure 9 Comparison of predicted and experimental beam patterns $2\alpha L = 0.25$ and 1, $k_0/k = 50$.

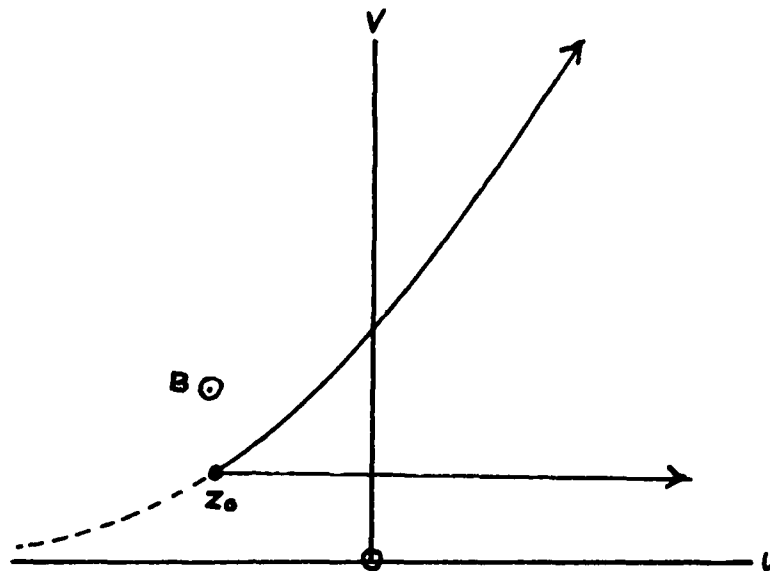


Fig. 2 - Contour Integration Paths

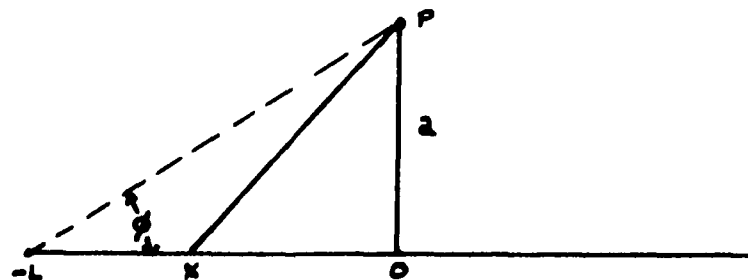


Fig. 1 - Geometry of Line Source

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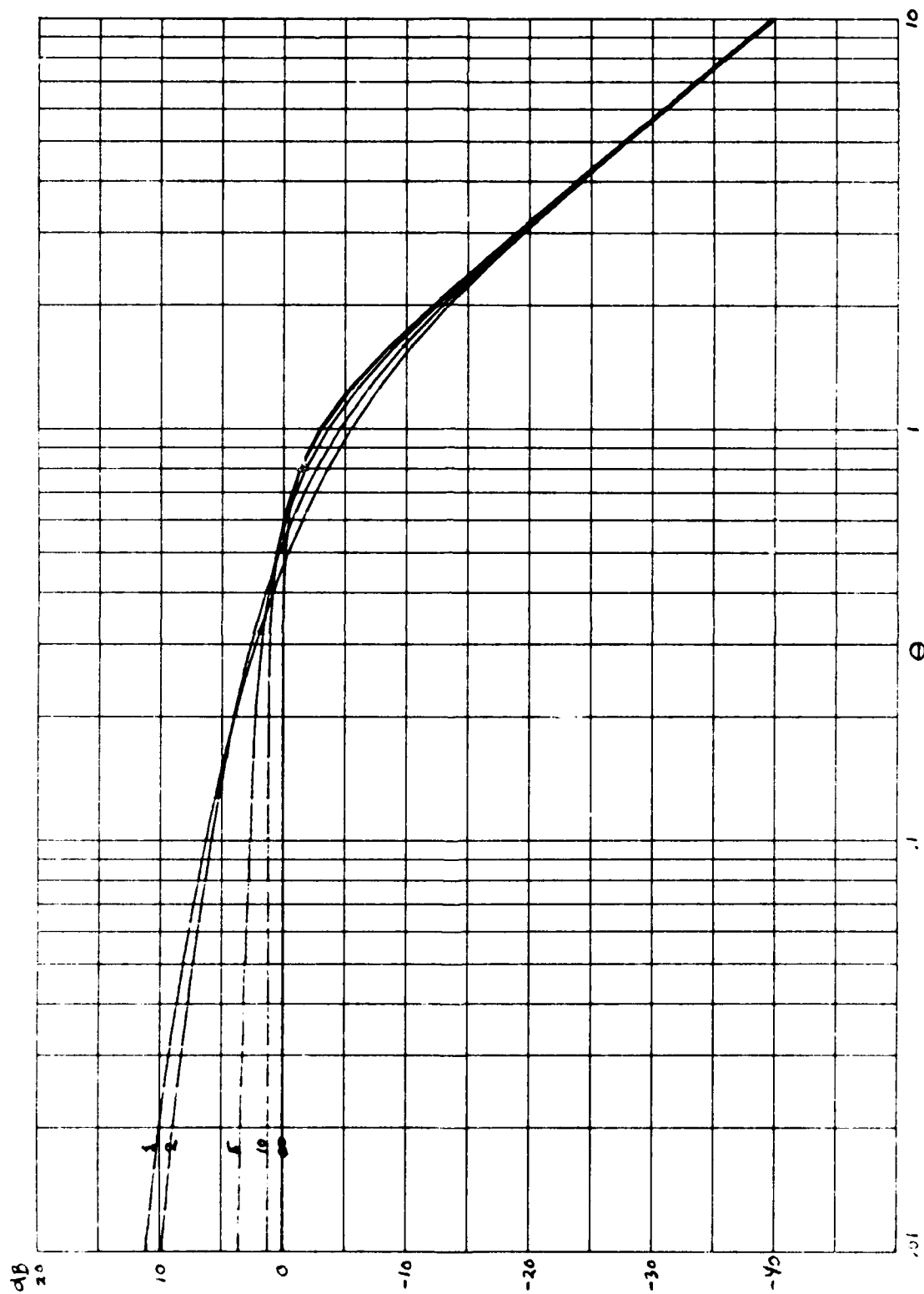


Fig. 2 - Beam Patterns

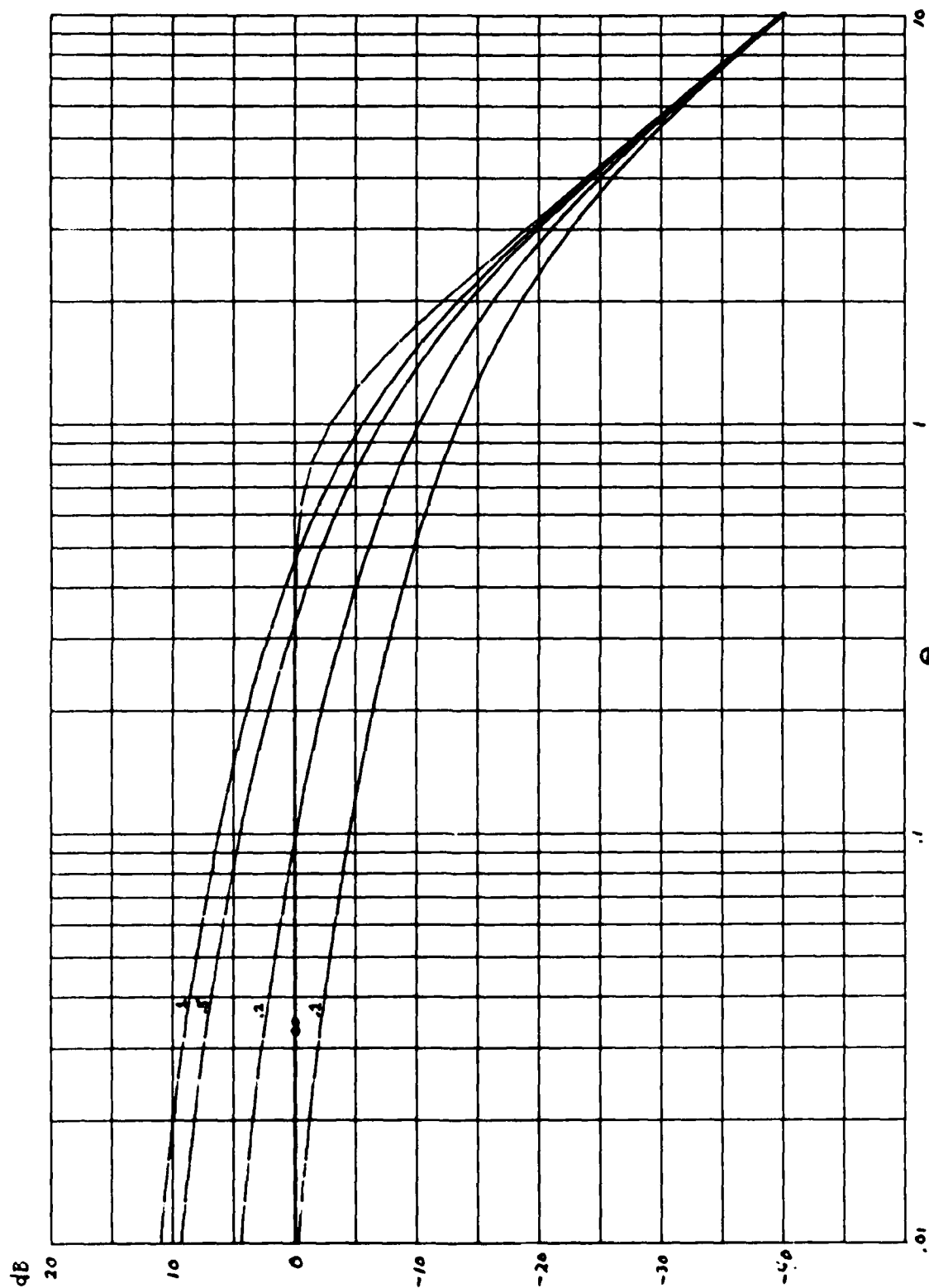


Fig. 4 - Beam Patterns $2dL = 1$ to .01
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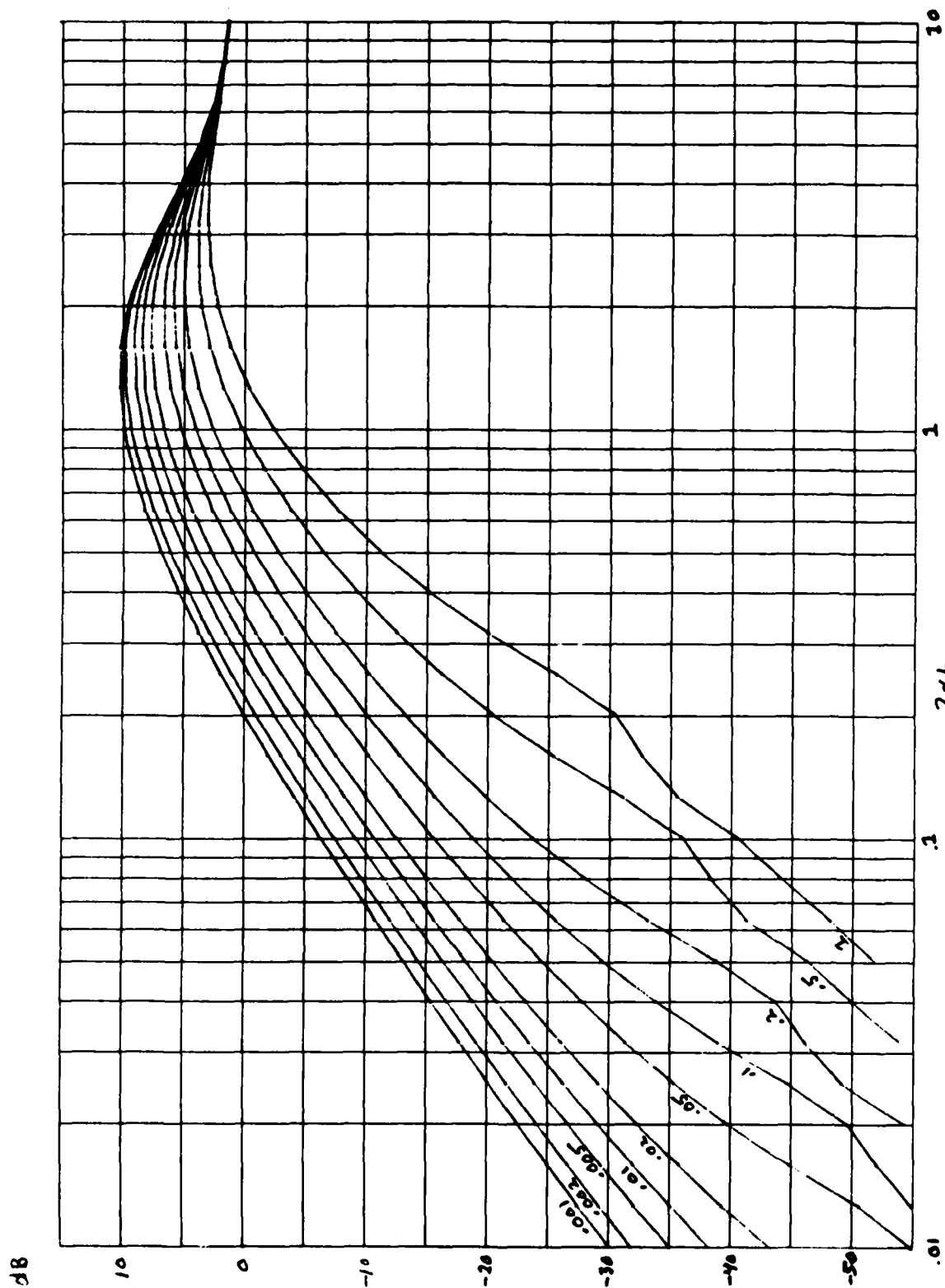


Fig. 5 - Source level relative to Westervelt vs. scaled distance. The parameter is the nearfield absorption number (plane wave model).

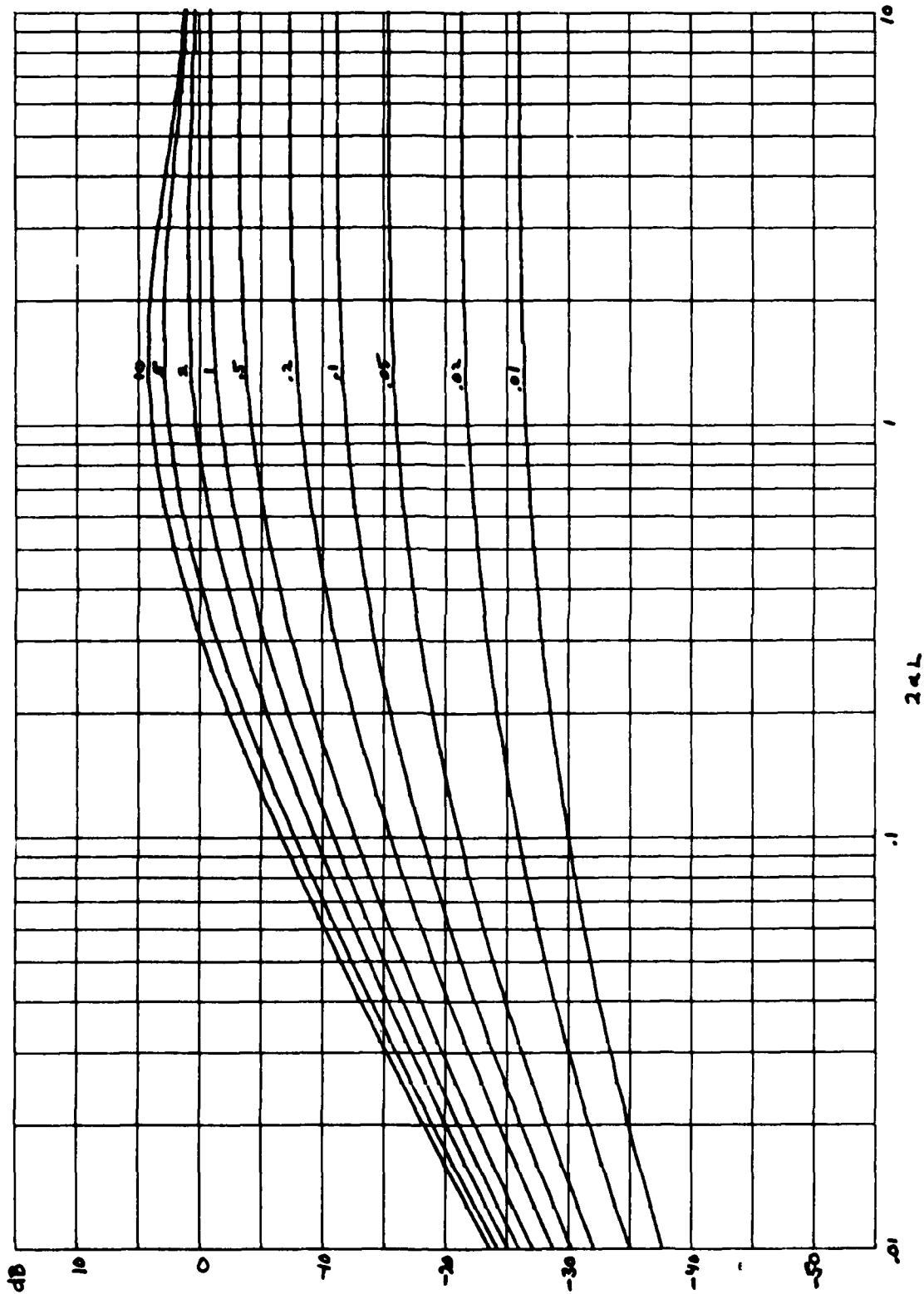


Fig. 6 - Source level relative to Westervelt vs. scaled distance. The parameter is the farfield absorption number (spherical wave model).

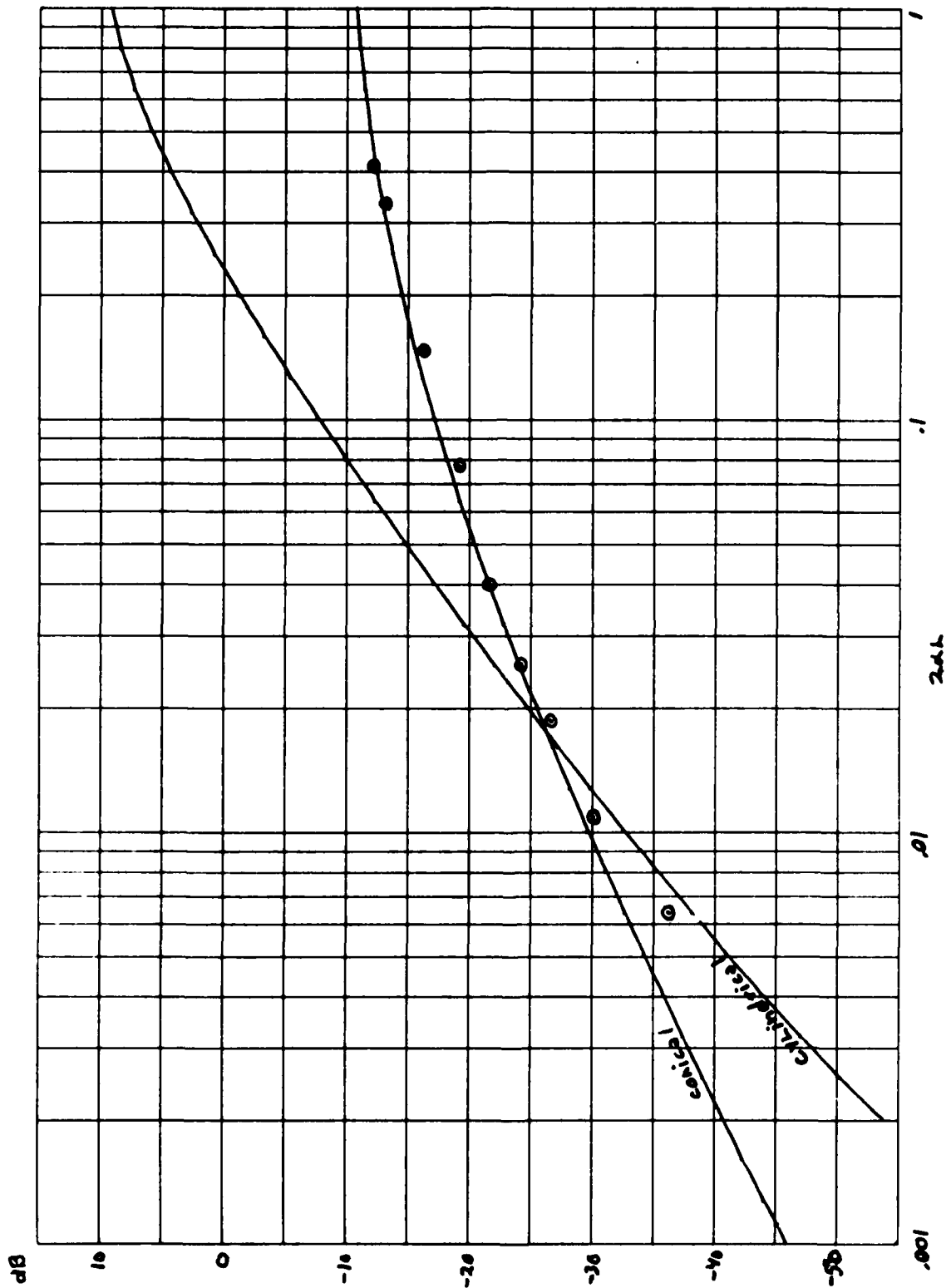


Fig. 7 - Comparison of predicted and experimental values of apparent source level vs. range $2\alpha R_0 = 0.0175$, $k_0/k = 7$

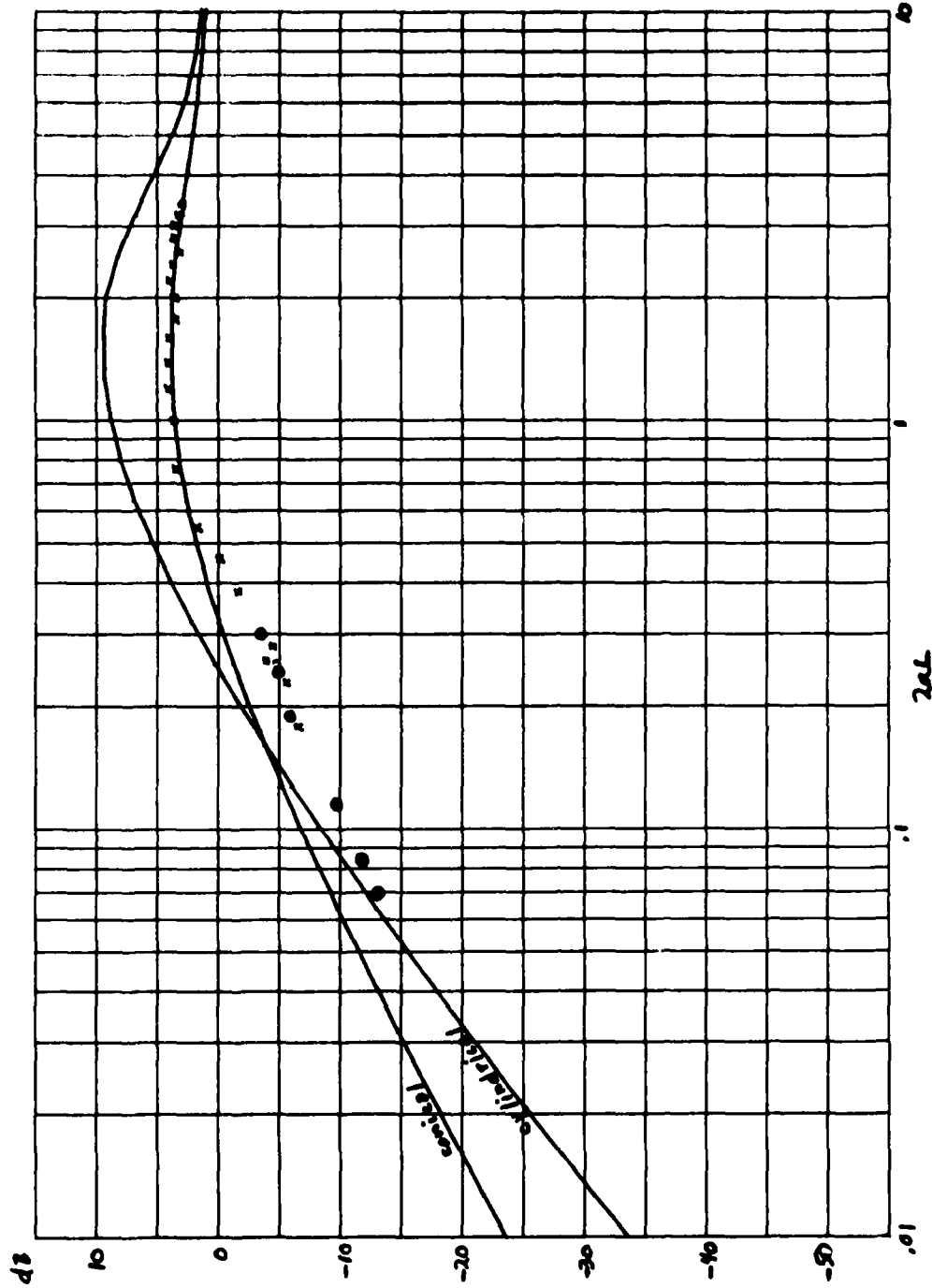


Fig. 8 - Comparison of predicted and experimental values of apparent source level vs. range
 $2\alpha R_0 = 0.17, k_0/k = 50$

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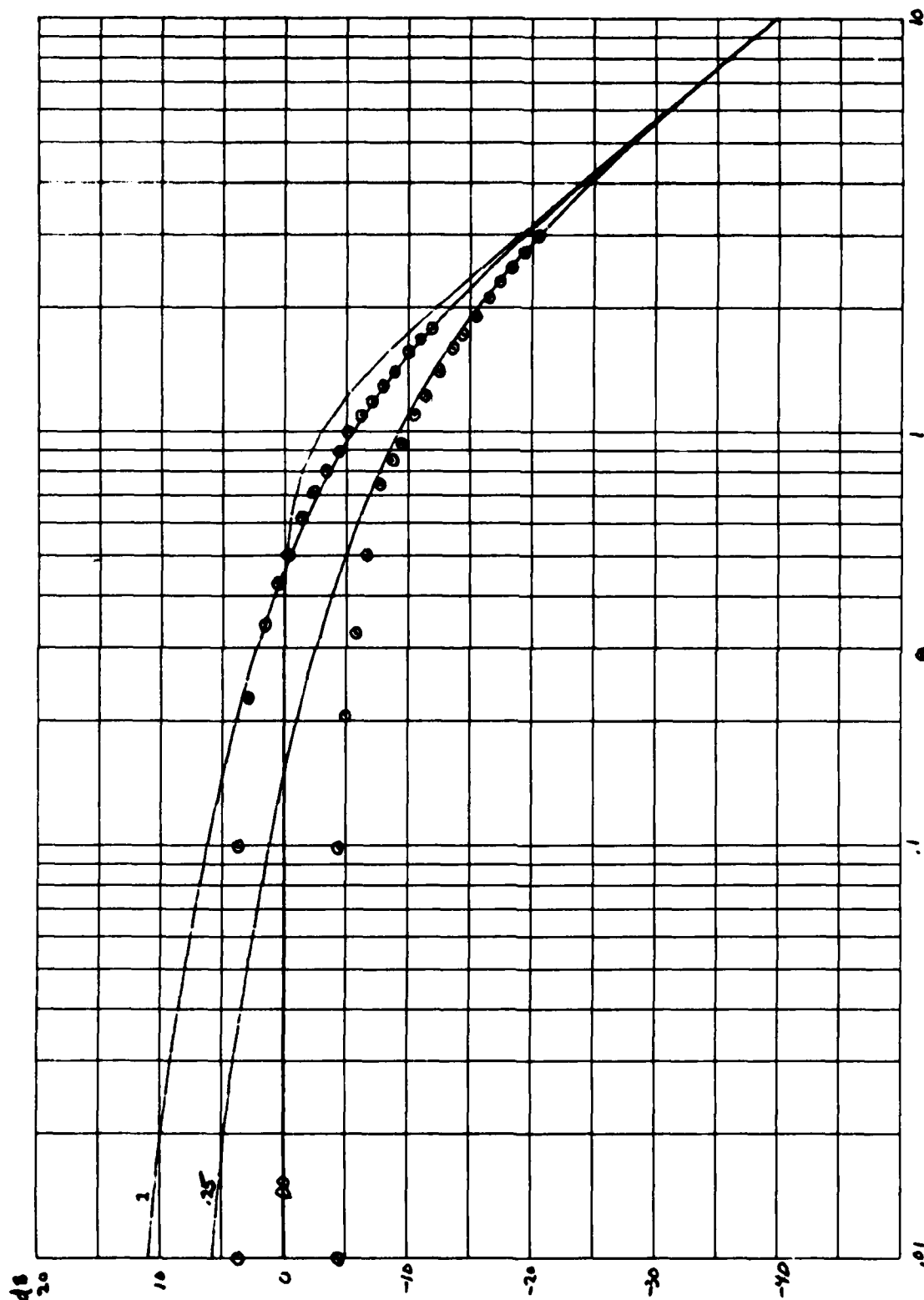


Fig. 9 - Comparison of predicted and experimental beam patterns 2 α $L = 0.25$ and 1, $k_0/k = 50$

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